10. Our concepts, applied to calculus

First draft of first section due Monday, November 22, 11:59 PM.

Final draft due Thursday, December 2, 11:59 PM. *No penalty for a late submission received by Friday, December 10, 11:59 PM.*

Formatting. The final draft of this assignment must be typed. The first draft need not be.

Remark. If you are using Microsoft Word, there are various options for typing up math symbols. There are also tutorials online for using TeX and LaTeX, though this is probably more time-intensive. *If you cannot produce a typed-up assignment in any reasonable manner, e-mail me and we can make an exception. Such an e-mail must be received by November 24th.*

Prompt. I want you to write a letter, where your target audience is a student who has just finished taking Calculus I, containing two sections:

(Section I) A section explaining what a function, a domain, a codomain, an injection, surjection, and bijection are.

You may assume that the only relevant sets are subsets of $\mathbb{R}$. Your essay must then treat the following examples:

(a) $f(x) = \tan(x)$, with domain given by $(-\pi/2, \pi/2)$ and codomain given by $\mathbb{R}$.
(b) $f(x) = 7x + 1$ with domain $\mathbb{R}$ and codomain $\mathbb{R}$.
(c) $f(x) = \sin(x)$ with domain $\mathbb{R}$ and codomain $[-1, 1]$.
(d) $f(x) = x^2$ with domain $\mathbb{R}$ and codomain $\mathbb{R}$.
(e) $f(x) = x^2$ with domain $\mathbb{R}$ and codomain $[0, \infty)$.
(f) $f(x) = x^2$ with domain $[0, \infty)$. and codomain $[0, \infty)$..

For each of the above examples, you must state whether the function is an injection, surjection, bijection (or none of these), and provide a proof of your claim.

(Note that some of the domains above may not be the domain you are always used to. So you may say, "for the purposes of this example, we will declare our domain to be the open interval from $\pi/2$ to $\pi/2$," and so forth.)

As stated above, the first draft of this first section is due Monday, November 22, 11:59 PM.

(Section II) A section about proofs by induction. After explaining what a proof by induction is, your essay must then state and prove (using induction) the following proposition:

Proposition (The Power Law). For every $n \geq 1$, $(x^n)' = nx^{n-1}$.

In proving this, you may explicitly state that you assume the student knows that $(x)' = 1$ and is familiar with the product rule (otherwise known as the Leibniz rule) for derivatives.

In your essay, you may motivate this proposition by saying it is an example of a proof by induction.

The idea. The idea is to produce an essay that may be helpful for you to review in the future, or may also have been helpful to you if someone had given it to you at the start of this semester. It may also be a way to motivate Calculus I students to see that they can jump right into Math 3330 if they wish.