Extra Credit 4.

You and your friend are at the natural number restaurant. The menu consists of natural numbers. You and your friend, due to budgetary constraints, will order exactly one menu item each.

Notation 3.1. Let $\mathbb{N} \times \mathbb{N}$ be the set of all orders that the two of you could place. For simplicity, if you requests a menu item (that is, a natural number) called a, and if your friend requests a menu item called b, let's write this order as (a, b).

Example 3.2. So for example, possible orders include $(0, 0)$ or $(1, 0)$ or $(0, 1)$ or $(13, 129)$, among (infinitely) many others. Put another way, $(0, 0) \in \mathbb{N} \times \mathbb{N}$ and $(1, 0) \in \mathbb{N} \times \mathbb{N}$ and $(0, 1) \in \mathbb{N} \times \mathbb{N}$ and $(13, 129) \in \mathbb{N} \times \mathbb{N}$.

Remark 3.3. Note that (a, b) is a different "order" than (b, a); also note that it is possible for you and your friend to request the same thing, so $\mathbb{N} \times \mathbb{N}$ contains elements that look like $(1, 1)$, $(3, 3)$, $(189, 189)$, et cetera.

Exhibit a bijection between $\mathbb{N}$ and $\mathbb{N} \times \mathbb{N}$.