# Lecture 13

# In-class exercises

Hiro is gone Monday and Wednesday. You will have a guest professor, Stewart Welsh, present to answer any math questions you may have.

On Monday and Wednesday, please work on the following problems (in any order you choose).

## 13.1 Big questions

**Exercise 13.1.1.** Is there such a thing as "a set of all sets?" That is, a set whose elements are all conceivable sets?

We saw Cantor's theorem (no set admits a bijection to its power set) last time. If there were a set of all sets, what would Cantor's Theorem have to say about it?

**Exercise 13.1.2.** We saw early on in class that there is no such thing as a 'biggest number.'

Is there such a thing as a 'biggest set?'

For the purposes of this exercise, let's say that a set $Y$ is bigger than a set $X$ if there exists an injection from $X$ to $Y$, but there does not exist a surjection from $X$ to $Y$.

## 13.2 Practice

**Exercise 13.2.1.** Suppose that $f : X \to Y$ is a surjection and $g : Y \to Z$ is a surjection. Show that the composition $g \circ f$ is a surjection.

Remember that "injection" and "surjection" can intuitively say things about size, but size is *not* helpful to the proofs we conduct in this class. Instead, we must use the definitions of injection and surjection.

**Exercise 13.2.2.** Suppose that $f : X \to Y$ is an injection and $g : Y \to Z$ is an injection. Show that the composition $g \circ f$ is an injection.

**Exercise 13.2.3.** Suppose that $f : X \to Y$ is a bijection. Construct a bijection from $Y$ to $X$.

**Exercise 13.2.4.** Given a set $X$, show that there is always an injection from $X$ to $\mathcal{P}(X)$ (the power set) by constructing one.