Lecture 19

Exercises: And/Or/Negation, implication, Truth Tables

Exercise 19.0.1. Let A and B be a collection of statements, and where each statement in A or B is known to be either true or false (but not both).

Let $T(A)$ be the subset of A and consisting of all elements of A that are true, and $F(A)$ the subset of A consisting of false statements. Likewise define $T(B)$ and $F(B)$.

Which of the following is true?

(a) If $(p, q) \in T(A) \times T(B)$, then "$p$ and q" is a true statement.

(b) If $(p, q) \in F(A) \times T(B)$, then "$p$ and q" is a true statement.

(c) If $(p, q) \in T(A) \times F(B)$, then "$p$ and q" is a true statement.

(d) If $(p, q) \in F(A) \times F(B)$, then "$p$ and q" is a true statement.

(e) If $(p, q) \in T(A) \times T(B)$, then "$p$ or q" is a true statement.

(f) If $(p, q) \in F(A) \times T(B)$, then "$p$ or q" is a true statement.

(g) If $(p, q) \in T(A) \times F(B)$, then "$p$ or q" is a true statement.

(h) If $(p, q) \in F(A) \times F(B)$, then "$p$ or q" is a true statement.

(i) $A \times B = (T(A) \times T(B)) \bigcup (T(A) \times F(B)) \bigcup (F(A) \times T(B)) \bigcup (F(A) \times F(B))$.

Table 19.1:

p	q	p $\implies$ q	$\neg$ q	$\neg$ p	$\neg q \implies \neg p$
T	T				
T	F				
F	T				
F	F				

Exercise 19.0.2. You have been given the truth tables for $\implies$ and $\neg$. Using these, fill out Table 19.1. What do you notice about the third column and the last column?

Exercise 19.0.3. For each statement p below, write out (a statement equivalent to) $\neg p$.

(a) $a \geq 0$.

(b) $ab = 0$.

(c) For every person x, there exists a person y with whom they can live happily ever after.

(d) Every happy family is happy for the same reason.

(e) Every dog has its day.

(f) Every situation has a silver lining.

(g) For every situation, there exists a silver lining.

(h) For every $\epsilon > 0$, there exists a $\delta > 0$ so that $0 < |x - a| < \delta \implies |f(x) - L| < \epsilon$.[1]

(i) For every $\alpha \in \mathcal{A}$, x is an element of U_α.[2]

(j) For some $\alpha \in \mathcal{A}$, x is an element of U_α.[3]

[1] Recall this is what it means for a function f to have a limit L at a.
[2] Recall this is what it means for x to be in the intersection $\bigcap_{\alpha \in \mathcal{A}} U_\alpha$.
[3] Recall this is what it means for x to be in the union $\bigcup_{\alpha \in \mathcal{A}} U_\alpha$.