Lecture 20

Exercises: Converse, Contrapositive, Equivalence

Exercise 20.0.1. Suppose that A and A' are compound statements. (For example, A could equal $(p \wedge q) \vee q$.) Likewise, let B and B' be compound statements.

Show that if A is logically equivalent to A', and if B is logically equivalent to B', then $A \implies B$ is logically equivalent to $A' \implies B'$.

Exercise 20.0.2. Can you think of compound statements A, A', B, B', all depending on exactly two statements p and q, so that $A \wedge B$ is not logically equivalent to $A' \wedge B'$, but where $A \vee B$ is logically equivalent to $A' \vee B'$?

Exercise 20.0.3. Write out the converse to each of the following statements:

1. If n is an even natural number, then n is a natural number divisible by 2.

2. If R is a shape with four sides, then R is a rectangle.

3. If f is differentiable, then f is continuous.

4. If you get a 100 on all your assignments for this class, you get an A in the class.

For each of the above, compare the truth value of the original statement to the truth value of the converse. For which statements do we have the same truth values?

Exercise 20.0.4. Write out the converse to each of the following statements:

1. If $x \in A$ and $x \in B$, then $x \in A \cap B$.
2. If $x \in A$ and $x \notin B$, then $x \in A \setminus B$.
3. If $x \in \mathbb{R}$ and $x = \sqrt{2}$, then $x \notin \mathbb{Q}$.
4. Let x be a real number. If x can be written as a fraction of two integers, then the decimal expansion of x eventually repeats a finite string of digits.

For each of the above, compare the truth value of the original statement to the truth value of the converse. For which statements do we have the same truth values?