Marak Guest talk. ①

2d Super Cnfml Algebra, $N=(2,2)$

has • an automphm

• interpreth as a sigma model $\longrightarrow X$, $\hookrightarrow X^\vee$.

So automphm $\Rightarrow \exists\ X, X^\vee$ w/ same thy.

CY 3-folds

$$\text{IIA} \left\{ \underbrace{X \times \mathbb{R}^{1,3}}_{\text{4d}, N=2 \text{ Thy.}} \qquad \begin{matrix} X^\vee \\ \times \\ \mathbb{R}^{1,3} \end{matrix} \right\} \text{IIB}$$

signature 1,3

Central charge comes from possibility of 2 central exchge.

$\exists$ nice notion of deformtn of SCA.

Think of as $X, X^\vee$ carry in families.

$X_t \qquad X^\vee_u$ — coordinate on $\mathbb{C}$ struct moduli space

coordinats on mathmatically weird space of Kähler structs, birational transfrmtns, Crepant resolution of moduli.

Early 90s story; identfy t, u.

Kontsevich: $D^b Coh(X_\tau) \cong Fuk(X_u^\vee)$.

categories, not definitions.

objects of cat = BPS states.

particular set of distinguished states in 4D theory; (on $\mathbb{R}^{1,3}$) they don't know whether they came from A model or B model.

Though you define $\mathbb{C}$ sheaf of $X_u^\vee$, Fuk won't change. (So B model deforms of A model.)

BPS states can be rep. as BPS quiver representations.

$$D^b Coh(X_\tau) \cong Fuk(X_u^\vee)$$
$$\cup\!\!\!| \qquad \cup\!\!\!|$$
$$Rep(Quiver) \quad \text{cones}$$

change of Stab condition is change of core exchanges of algebra; so change in u won't change Fuk, but it changes stability conditions

Not known whether these quivers actually generate whole category

What are 4d, $N=2$ Gauge theories?

Need to specify: • Gauge group G, rank r

• moduli space = Cartan subalgebra of G, $U(1)^r$

(i.e., gauge group is broken to maximal Cartan subalg).

u local coord on Cartan subalg.

This translates to moduli space of $\mathbb{C}$ structures, for example.

• Γ lattice of rank $2r$.

(ex. $H_3(X_u^\vee; \mathbb{Z})$.)

$\gamma \in \Gamma$ is called a charge, or charge vector.

~~$\gamma \in e$~~ $\gamma = (e^a, m_a)$

electric magnetic.

Why $\dim \mathrm{Stab}\, \mathcal{C} = 2 \dim$ Physical space of stab?

Really only $r = \frac{1}{2} r$ depends on physics of theory

Only half elements of $H_3(X; \mathbb{Z})$ correspond to actual $\mathbb{C}$ deformations.

• Symp pairing $\langle , \rangle : \Gamma \times \Gamma \longrightarrow \mathbb{Z}$

$\langle \gamma_i, \gamma_j \rangle$

Mukai pairing

$\langle \mathcal{E}, \mathcal{F} \rangle = \int_X ch(\mathcal{E}) ch(\mathcal{F}) \sqrt{td X}$

or

wedge product on H_3.

• Z central charge, depends on u.

$$Z_u : \Gamma \longrightarrow \mathbb{C}.$$

• BPS bound: Label particles by $\gamma \in \Gamma$.

Mass of particle, γ, is $\geq$ to $Z(\gamma)$.

$$M_\gamma \geq |Z_u(\gamma)|$$

Same other fun

e.g.,

$$\left|\int_L \Omega\right| \leq \int_L |\Omega|.$$

equal only when L is special!

$|Z|$

M_γ.

• BPS state γ are the ones satisfying

$$M_\gamma = |Z_u(\gamma)|.$$

Decay: $\gamma = \gamma_1 + \gamma_2$.

For physical decay, we also need

$$M_\gamma = M_{\gamma_1} + M_{\gamma_2}$$ (mass and charge conserved)

by $\triangle$ inequality, $|Z(\gamma)| \leq |Z(\gamma_1)| + |Z(\gamma_2)|$

no decay.

decay possible

walls are

Wall crossing, loci of codim 1

on one side of wall, all particles exist

on other side, only smaller ones exist

BPS Quiver:

· γ BPS particle then $-\gamma$ should be in spectrum of BPS particles,

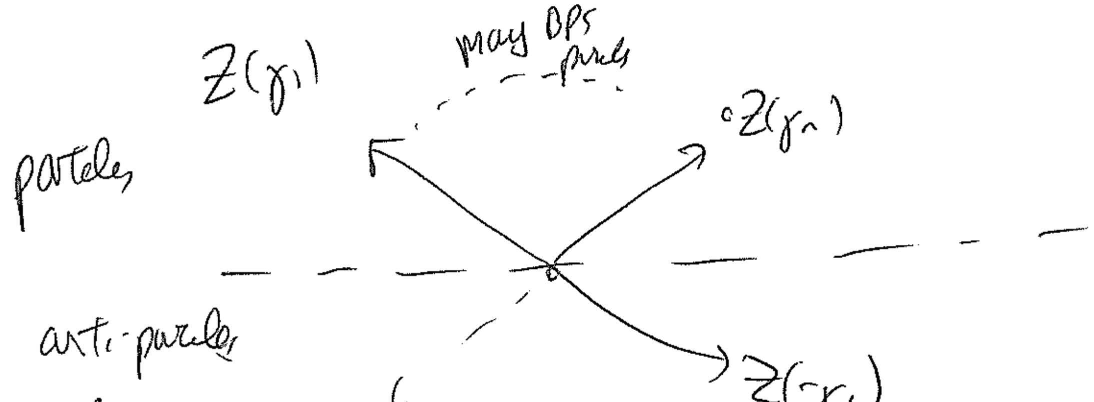
$$\gamma \in BPS \Rightarrow -\gamma \in BPS$$

· which is particle, which is anti-particle?
Kind of a random choice. How to make precise?

Choose half of BPS states.

t-structure on category! (Choosing half a category.)

To do this, use central charge.



gives cone of particles, and
cone of anti-particles

Changing choice can tell us everything about category.

· To make this choice, choose basis of BPS states,

$\{\gamma_1, \dots, \gamma_{2r}\} \in \Gamma$ such that:

$\forall \gamma \in$ {particles} (NOT anti-particles), $\gamma = \sum a_i \gamma_i$, $a_i \in \mathbb{Z}_{\geq 0}$.

Quiver: A diagram associated to basis

- node to every $\gamma_i \in$ basis
- arrows A_{ij}^{a} between γ_i and γ_j

# of arrows computed by $\langle \gamma_i, \gamma_j \rangle$.

positive: i to j
negative: other way.

Diagrammatic way to encode spectrum.

? Given $\gamma = \sum_{i=1}^{2r} n_i \gamma_i$, is $\gamma \in \{\text{particles}\}$?

This will be answered next time; not every $\gamma \in \Gamma$ will be stable. Ex: Not every Lagrangian is special. How does this depend on u? Next time