Zgox (29/10)

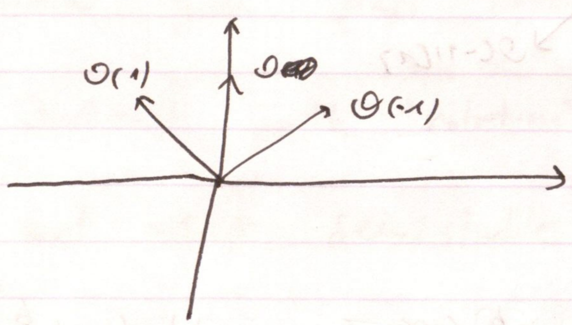
Example of wall crossing.

Recall: stability condition on $Coh(\mathbb{P}^1)$ rotated.

determined by where we send

$\mathcal{O}$ and $\mathcal{O}(-1)[1]$

equiv to left $k(\bullet \overset{f}{\underset{g}{\rightrightarrows}} \bullet)$-modules where $ke_1 \rightrightarrows ke_2 \oplus kf \oplus kg$



$\xrightarrow{\text{rotate}}$ equiv to $\mathrm{Rep}(\bullet \rightrightarrows \bullet)$ (path algebra of quiver rep)

$D^b Coh(\mathbb{P}^1) \longmapsto D^b \mathrm{Rep}(\bullet \rightrightarrows \bullet)$

Hom cochain complex

$E \longmapsto \mathrm{Hom}^\bullet(\mathcal{O} \oplus \mathcal{O}(1), E)$

$\mathcal{O} \longmapsto k \langle id_{\mathcal{O}} \rangle$

$\mathcal{O}(-1)[1] \longmapsto \Sigma\, k \langle id_{\mathcal{O}(-1)} \rangle$

$End(\mathcal{O} \oplus \mathcal{O}(1)) = k\cdot id_{\mathcal{O}} \oplus k\cdot id_{\mathcal{O}(1)} \oplus k\cdot x_0 \oplus k\cdot x_1$

where x_0, x_1 are maps $\mathcal{O} \overset{x_0}{\underset{x_1}{\rightrightarrows}} \mathcal{O}(1)$.

$$k\left(\underset{1}{\bullet} \overset{f}{\underset{g}{\rightrightarrows}} \underset{2}{\bullet}\right)$$

do this first

$A \backslash M$	e_1	e_2	f	g
e_1	e_1	0	0	0
e_2	0	e_2	f	g
f	f	0	0	0
g	g	0	0	0

$\mathrm{End}(\mathcal{O} \oplus \mathcal{O}(1))$

$A \backslash M$	$id_{\mathcal{O}(1)}$	$id_{\mathcal{O}}$	x_0	x_1
$id_{\mathcal{O}(1)}$	$id_{\mathcal{O}(1)}$	0	0	0
$id_{\mathcal{O}}$	0	$id_{\mathcal{O}}$	x_0	x_1
x_0	x_0	0	0	0
x_1	x_1	0	0	0

do this first

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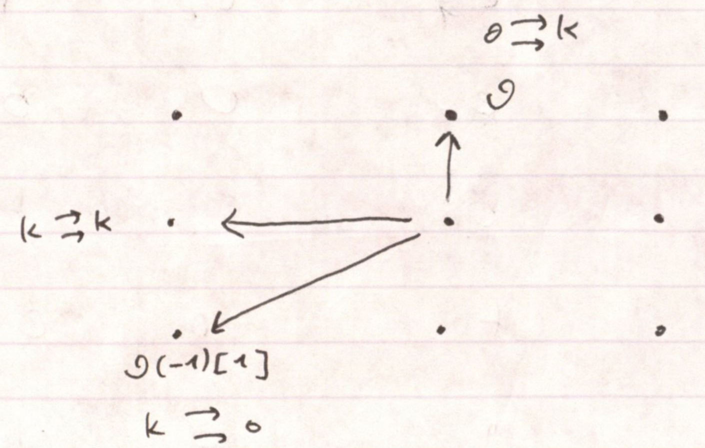
$$k \cdot id_{\mathcal{O}(1)} \rightrightarrows k \cdot id_{\mathcal{O}} \oplus k x_0 \oplus k x_1$$

So $\quad \mathcal{O} \longleftrightarrow 0 \rightrightarrows k$

$\mathcal{O}(-1)[1] \longleftrightarrow k \rightrightarrows 0$



$$K_0(D^b \operatorname{Coh}(\mathbb{P}^1)) \longrightarrow K_0(D^b \operatorname{Rep}(\cdot \rightrightarrows \cdot))$$

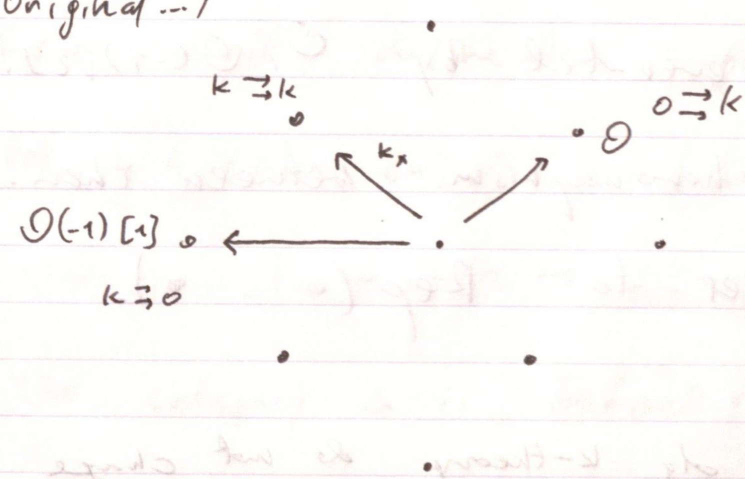
(degree, rank) $\longmapsto$ (d_1, d_2) dimensions in quiver

i.e. $(0,1) \longmapsto (0,1)$

 $(1,0) \longmapsto (1,1)$

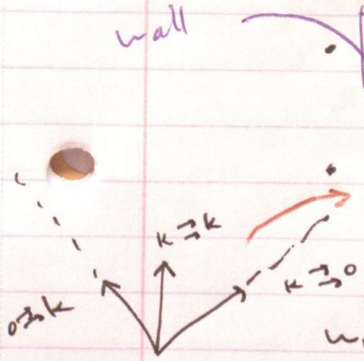
so the matrix is $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

So now if we'll do the rotation (different from last time and probably from the original...)



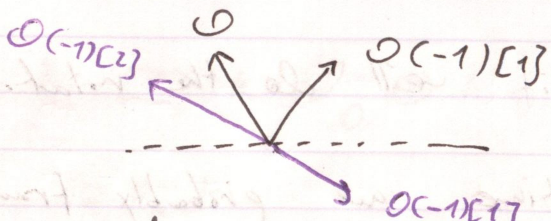
So after rotation, the heart is generated by $\mathcal{O}$, $\mathcal{O}(-1)[1]$. Let $z_0 = \phi(Z(\mathcal{O}))$, $z_{-1} = \phi(Z(\mathcal{O}(-1)[1]))$. The last time we've said that depending on the case:

- $z_0 < z_{-1}$ equiv to standard situation by reparam of the plane.
- wall $\boxed{z_0 = z_{-1}}$ every nonzero obj is semistable.
- $z_0 > z_{-1}$ only semistable obj are multiples of $\mathcal{O} \rightrightarrows k$, $k \rightrightarrows \mathcal{O}$.



we get different semistable objects.

If $z_0 > z_1$



and shift z_{-1} <u>below real axis.</u> Then the new heart is generated by $\mathcal{O}$, $\mathcal{O}(-1)[2]$. There are no homomorphism between them. This correspondes to Rep(• •).

<u>Comment.</u> Higher alg K-theory do not change when you change the heart. So this is a good way to compute.

<u>Comment.</u> The category of spectra is an overkill ... but here we have a simple example for a ~~[illegible]~~ such that .