TALBOT SEMINAR

AGNÈS BEAUDRY

1. INTRODUCTION

My main references are [1], [2] and [3]. The goal of this talk is to define Morava E-theory E_n, and to introduce a spectral sequence, due to Morava

$$H^s(\mathbb{G}_n, (E_n)_t X) \Longrightarrow \pi_{t-s} L_{K(n)} X.$$

This spectral sequence, often called the *homotopy fixed points* spectral sequence is the key tool in many of the $K(n)$-local computations.

2. MORAVA E-THEORY FROM THE POINT OF VIEW OF LANDWEBER EXACTNESS

Let ζ be a primitive $p^n - 1$ root of unity and

$$W(\mathbb{F}_{p^n}) = \mathbb{Z}_p[\zeta].$$

The ring $W(\mathbb{F}_{p^n})$ is a complete local ring with maximal ideal (p), and

$$W(\mathbb{F}_{p^n})/(p) \simeq \mathbb{F}_{p^n}.$$

Define a graded ring

$$(E_n)_* = W(\mathbb{F}_{p^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}],$$

with $|u| = -2$ and $|u_i| = 0$. Recall that

$$BP_* = \mathbb{Z}_{(p)}[v_1, v_2, \ldots],$$
$$|v_i| = 2(p^n - 1).$$

The universal p-typical formal group law F_{BP} is defined by

$$[p]_{F_{BP}}(x) = px +_{F_{BP}} v_1 x^p +_{F_{BP}} \ldots +_{F_{BP}} v_n x^{p^n} +_{F_{BP}} \ldots.$$

There is a map

$$F : BP_* \to (E_n)_*$$

$$F(v_i) := \begin{cases} u_i u^{1-p^i} & 0 < i < n; \\ u^{1-p^n} & i = n; \\ 0 & i > n. \end{cases}$$

This defines a graded p-typical formal group law F with p-series

$$[p]_F = px +_F u_1 u^{1-p} x^p +_F \ldots +_F u_{n-1} u^{1-p^{n-1}} x^{p^{n-1}} +_F u^{1-p^n} x^{p^n}.$$

Since u is invertible, the sequence $(p, u_1 u^{1-p}, \ldots, u^{1-p^{n-1}}, u^{1-p^n}, 0, 0, \ldots)$ is regular, so $(E_n)_*$ is Landweber exact and it defines a ring spectrum E_n.

Using u, we can shift F to degree zero,

$$F_{E_n}(x, y) := u^{-1} F(ux, uy).$$

Definition 1. *The spectrum E_n is called Morava E-theory. It is a complex oriented 2 periodic ring spectrum. Its formal group law is p-typical, defined by*

$$(E_n)_* = W(\mathbb{F}_{p^n})[[u_1, \ldots, u_{n-1}]][u^{\pm 1}]$$

$$[p]_{F_{E_n}}(x) = px +_{F_{E_n}} u_1 x^p +_{F_{E_n}} \ldots +_{F_{E_n}} u_{n-1} x^{p^{n-1}} +_{F_{E_n}} x^{p^n}.$$

The ring $(E_n)_*$ is a complete local ring with maximal ideal $(p, u_1, \ldots, u_{n-1})$ and residue field $\mathbb{F}_{p^n}$

Recall that

$$K(n)_* = \mathbb{F}_p[v_n^{\pm 1}]$$

$$[p]_{F_{K(n)}}(x) = v_n x^{p^n}.$$

Let $(K_n)_* = \mathbb{F}_{p^n}[u^{\pm 1}]$ with $|u| = -2$ and define an extension

$$K(n)_* \hookrightarrow (K_n)_*$$

by sending

$$v_n \mapsto u^{1-p^n}.$$

This is a faithfully flat extension. Hence there is a homology theory

$$(K_n)_* X = K(n)_*(X) \otimes_{K(n)_*} (K_n)_*.$$

These theories have weakly equivalent localization functors, hence the same local categories. So we can work with K_n instead of $K(n)$. The spectrum K_n is has the advantage of being 2-periodic. We can use u to shift the formal group law to degree zero,

$$u^{-1} F_{K_n}(ux, uy).$$

This gives the non-graded Honda formal group law Γ_n over $\mathbb{F}_{p^n}$. It is p-typical, defined by

$$[p]_{\Gamma_n}(x) = x^{p^n}.$$

Let

$$\pi : (E_n)_0 \to \mathbb{F}_{p^n},$$

be the projection modulo the maximal ideal $(p, u_1, \ldots, u_{n-1})$. Then

$$\pi_*(F_{E_n}) = \Gamma_n.$$

So F_{E_n} is a lift of Γ_n over $(E_n)_0$.

This suggest the relation between $\mathbb{G}_n$ and E_n. Morava E-theory E_n arises from the Honda formal group law through Lubin-Tate *deformation theory*.

3. Morava E-theory and Deformations

Let k be a perfect field of characteristic $p > 0$. Let Γ be a height n formal group law over k.

Definition 2. *Let B is a complete Noetherian local ring with maximal ideal $\mathfrak{m}$ and projection $\pi : B \to B/\mathfrak{m}$. A deformation of Γ over B is a pair (G, i) which satisfies:*

- *G is a formal group law over B.*
- *$i : k \to \pi(B) = B/\mathfrak{m}$ is an isomorphism.*
- *$i^* \Gamma = \pi^* G$.*

In other words, G is a lift of Γ to B.

A morphism between two deformations is defined as follows:

Definition 3. ⋆-isomorphisms *Given two deformations (G_1, i_1) and (G_2, i_2) over B, a morphism can exist only if $i_1 = i_2$. Then a morphism is given by an* isomorphism *of formal group laws $f : G_1 \to G_2$, which reduces to the identity modulo $\mathfrak{m}$, i.e,*

$$\pi^*(f) : i_1^*(\Gamma) = \pi^*(G_1) \xrightarrow{id} \pi^*(G_2) = i_2^*(\Gamma).$$

The deformations of Γ over B form a category $\mathrm{Def}_\Gamma(B)$ with

$$\mathrm{ob}(\mathrm{Def}_\Gamma(B)) = \{(G, i)\}$$
$$\mathrm{hom}(\mathrm{Def}_\Gamma(B)) = \{\star - \text{isomorphisms}\}.$$

The category $\mathrm{Def}_\Gamma(B)$ is a groupoid in the sense that all its morphisms are invertible. This construction defines a functor. Let Λ be the category of complete Noetherian local rings with continuous ring homomorphisms. Then

$$\mathrm{Def}_\Gamma(-) : \Lambda \to \text{Sets}.$$

Definition 4. *A* universal deformation *consists of:*

- *a complete local ring $R(k, \Gamma)$ such that $R(k, \Gamma)/\mathfrak{m} = k$.*
- *a deformation $(F(k, \Gamma), id)$ over $R(k, \Gamma)$ such that the following diagram can be filled by a unique F*

$$
\begin{array}{ccc}
R(k, \Gamma) & \xdashrightarrow{\ f\ } & B \\
\downarrow & & \downarrow{\scriptstyle \pi} \\
R(k, \Gamma)/\mathfrak{m} = k & \xrightarrow{\ i\ } & B/\mathfrak{m}
\end{array}
$$

so that

$$(G, i) \simeq^\star (f_* F(k, \Gamma), i),$$

via a unique ⋆-isomorphism.

Theorem 1. Lubin-Tate Theorem *Let $\mathrm{Def}_\Gamma(B)_i$ be the deformations of the form (G, i) for a fixed i. There are isomorphisms*

$$\pi_0(\mathrm{Def}_\Gamma(B)_i) \simeq \mathfrak{m}^{n-1}$$
$$\pi_1(\mathrm{Def}_\Gamma(B), (G, i)) \simeq \{1\}.$$

The functor $\pi_0(Def_\Gamma(-)_i)$ is representable in the sense that there exists a universal deformation $R(k, \Gamma)$ so that

$$\mathrm{Hom}^c(R(k, \Gamma), B) \simeq \pi_0(Def_\Gamma(B)).$$

Remark 1. *There is a non-canonical isomorphism*

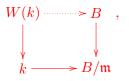$$R(k, \Gamma) \simeq W(k)[[u_1, \ldots, u_{n-1}]]..$$

If Γ is p-typical, then one can choose $F = F(k, \Gamma)$ with p-series

$$[p]_F(x) = x +_F u_1 x^p +_F \ldots +_F u_{n-1} x^{p^{n-1}} +_F x^{p^n}.$$

For Γ_n over $\mathbb{F}_{p^n}$, this implies that $(E_n)_0$ and F_{E_n} is a universal deformation of Γ_n, that is,

$$(E_n)_0 = R(\mathbb{F}_{p^n}, \Gamma_n).$$

Proof. I outline the idea. The Witt vectors $W(k)$ have the universal property that there is a lift for any complete local ring B,

$$
\begin{array}{ccc}
W(k) & \dashrightarrow & B \\
\downarrow & & \downarrow \\
k & \longrightarrow & B/\mathfrak{m}
\end{array} \quad,
$$

where the maps are continuous ring maps. So, $R(k,\Gamma)$ must be a $W(k)$ algebra. For $k = \mathbb{F}_{p^n}$, the local ring B contains the Teichmüller lifts of the $p^n - 1$ roots of unity. The ring B is p-complete, hence is a $\mathbb{Z}_p$-algebra. That is all the structure that complete local rings over $\mathbb{F}_{p^n}$ support. Hence

$$W(\mathbb{F}_{p^n}) \simeq \mathbb{Z}_p[\zeta].$$

Suppose that Γ is p-typical. Then

$$[p]_\Gamma(x) = a x^{p^n} + \dots$$

for some unit $a \in k$. Let $\overrightarrow{a} = (a_1, \dots, a_{n-1}) \in \mathfrak{m}^{n-1}$. The p-typical formal group law over B defined by

$$[p]_{F_{\overrightarrow{a}}}(x) = px +_{F_{\overrightarrow{a}}} a_1 x^p +_{F_{\overrightarrow{a}}} \dots +_{F_{\overrightarrow{a}}} a_{n-1} x^{p^{n-1}} +_{F_{\overrightarrow{a}}} a x^{p^n}.$$

Then $F_{\overrightarrow{a}}$ is a deformation of Γ over B. What needs to be proven is that, up to a **unique $\star$-isomorphism**, any deformation has this form. Then clearly

$$\mathrm{Hom}^c(R(k,\Gamma), B) \simeq \pi_0(\mathrm{Def}_\Gamma(B)_i)$$

In other words, you need to prove is that up to $\star$-isomorphism, you just need to *fill in the slots*. Although not difficult in itself, the proof is long and requires machinery. In particular, it uses Lazard's theorem on symmetric 2-cocycles. $\qquad\square$

3.1. **Landweber Exactness.** Any degree zero formal group law over a ring $R(k,\Gamma)$ can be given a grading. Let u be such that $|u| = -2$. Then

$$E(k,\Gamma)_* = R(k,\Gamma)[u^{\pm 1}]$$

is a graded ring and the formal group law

$$\overline{F}(k,\Gamma)(x,y) = u^{-1} F(k,\Gamma)(ux, uy),$$

is a degree -2 formal group law over $E(k,\Gamma)_*$. Hence it is classified by a map

$$\overline{F} : MU_* \to R(k,\Gamma)[u^{\pm 1}].$$

One can show that the functor

$$X \to MU_*(X) \otimes_{MU_*} R(k,\Gamma)[u^{\pm 1}].$$

is Landweber exact so that

    (1) There is a complex oriented 2-periodic ring spectrum $E(k,\Gamma)$ such that $\pi_0(E(k,\Gamma)) = R(k,\Gamma)$.
    (2) The degree zero formal group law associated to $E(k,\Gamma)$ is $F(k,\Gamma)$.

In this way, we recover

$$E_n = E(\mathbb{F}_{p^n}, \Gamma_n).$$

4. The Action of the Morava Stabilizer group $\mathbb{G}_n$ on E_n

Let $g \in \mathrm{Aut}(\Gamma)$,
$$g : \Gamma \to \Gamma.$$
The element g is a power series $g(x)$ with coefficients in k. Choose any lift $\widetilde{g}(x)$ of $g(x)$ to $R(k, \Gamma)$. Define a new formal group law by
$$\widetilde{F}(x, y) = \widetilde{g}^{-1} F(k, \Gamma)(\widetilde{g}(x), \widetilde{g}(y)).$$
Then $(\widetilde{F}, i)$ is a deformation of Γ over $R(k, \Gamma)$. This implies that there exists a unique ring isomorphism
$$g : R(k, \Gamma) \to R(k, \Gamma)$$
such that
$$g_* F(k, \Gamma) \simeq \widetilde{F}.$$
This defines an action of $\mathrm{Aut}(\Gamma)$ on the representing object $R(k, \Gamma)$.

The Galois group $\mathrm{Gal}(k/\mathbb{F}_p)$ also acts on $\mathrm{Def}_\Gamma(B)$. Let σ be the Fröbenius. The σ acts by
$$(G, i) \mapsto (G, i \circ \sigma).$$
If Γ is fixed by $\mathrm{Gal}(k/\mathbb{F}_p)$, then the induced action on $R(k, \Gamma)$ is just the action of the Galois group on $W(k)$. Hence we get a semi-direct product
$$\mathrm{Aut}(k, \Gamma) \simeq \mathrm{Aut}(\Gamma) \rtimes \mathrm{Gal}(k/\mathbb{F}_p).$$
There is a canonical way to extend the action of $\mathrm{Aut}(k, \Gamma)$ to $u \in E(k, \Gamma)_{-2}$ so that g can be realized through homotopy commutative maps of ring spectra.

Suppose that $g : E \to E$ is a stable ring operation on E. Evaluating g on a point gives a ring isomorphism
$$g^* : E^* \to E^*.$$
Let $x \in \widetilde{E}^0(\mathbb{C}P^\infty)$ be an orientation which restricts to u^{-1} under the restriction map
$$\widetilde{E}^0(\mathbb{C}P^\infty) \to \widetilde{E}^0(\mathbb{C}P^1) \simeq \widetilde{E}^0(S^2) = E_2.$$
The map
$$g^* : E^*(\mathbb{C}P^\infty) \to E^*(\mathbb{C}P^\infty)$$
sends x to a power $f(x) \in E^0[[x]]$ such that $f'(0)$ is a unit. By naturality, f gives an isomorphism of formal group laws
$$f : F_E \to \phi^* F_E.$$
Because $g^*(x) = f(x)$, the restriction of g^* to E_2 is given by
$$g^*(x + I^2) = f(x) + I^2 = f'(0)x + I^2,$$
so that,

(1) $$g^*(u^{-1}) = f'(0)u^{-1}.$$

5. The $K(n)$-local theory $(E_n)_* X$

Now recall that
$$\mathbb{S}_n = \mathrm{Aut}(\Gamma_n),$$
and
$$\mathbb{G}_n = \mathbb{S}_n \rtimes \mathrm{Gal}(\mathbb{F}_{p^n}/\mathbb{F}_p).$$

Then we have constructed an action of $\mathbb{G}_n$ on $(E_n)_*$.

Remark 2. *This action can be computed! Maybe not on the nose, but it can be approximated.*

Let $\mathcal{K}_n$ be the $K(n)$-local category. The functor

$$X \to BP_*(X) \otimes_{BP_*} (E_n)_* = \pi_*(E_n \wedge X)$$

does not preserve localizations, hence is not defined on $\mathcal{K}_n$. The category $\mathcal{K}_n$ has internal smash products and arbitrary wedges defined as follows. For X and Y in $\mathcal{K}_n$, let

$$X \wedge_{\mathcal{K}_n} Y = L_{K(n)}(X \wedge Y)$$

$$\bigvee_{\mathcal{K}_n} X_\alpha = L_{K(n)}\left(\bigvee X_\alpha\right).$$

We want $(E_n)_*$ to preserve this structure.

Definition 5. *Let X be a spectrum, then*

$$(E_n)_* X := \pi_* L_{K(n)}(E_n \wedge X).$$

This is not a homology theory, for example, it does not send arbitrary wedges to sums of abelian groups. But it has good properties which we explore here (in particular, it's computable!). First note that E_n is $K(n)$-local. So if X is finite, $E_n \wedge X$ is $K(n)$-local and

$$(E_n)_*(X) = \pi_*(E_n \wedge X).$$

In some good cases, $(E_n)_* X$ is a completion of $\pi_*(E_n \wedge X)$ with respect to

$$\mathfrak{m} = (p, u_1, \ldots, u_{n-1}).$$

Theorem 2. *If $K(n)_* X$ is concentrated in even degrees, then*

$$(E_n)_* X \simeq \pi_*(E_n \wedge X)_{\mathfrak{m}}^{\wedge}.$$

Further, $(E_n)_* X$ is the completion of a free $(E_n)_*$-module,

$$((E_n)_* X \simeq \bigoplus_\alpha \Sigma^{k_\alpha}(E_n)_*)_{\mathfrak{m}}^{\wedge}.$$

Now recall that there is an action of $\mathbb{G}_n$ on $(E_n)_*$. It can be obtained from a homotopy commutative maps of ring spectra $E_n \xrightarrow{g} E_n$. So they commute up to homotopy with the multiplication on E_n, i.e.,

$$
\begin{array}{ccc}
E_n \wedge E_n & \xrightarrow{g \wedge g} & E_n \wedge E_n \\
\downarrow & & \downarrow \\
E_n & \xrightarrow{\quad g \quad} & E_n
\end{array}
$$

This gives an action of $\mathbb{G}_n$ on $x \in (E_n)_* X$ via

$$(2) \qquad\qquad S \xrightarrow{\ x\ } E_n \wedge X \xrightarrow{g \wedge 1} E_n \wedge X \ .$$

In particular, if $a \in (E_n)_*$ and $x \in (E_n)_* X$, then there is a commutative diagram

$$
\begin{array}{ccccccc}
S & \longrightarrow & S \wedge S & \xrightarrow{a \wedge x} & E_n \wedge E_n \wedge X & \xrightarrow{m \wedge 1} & E_n \wedge X \\
& & & & \downarrow{\scriptstyle g \wedge g \wedge 1} & & \downarrow{\scriptstyle g \wedge 1} \\
& & & & E_n \wedge E_n \wedge X & \xrightarrow{m \wedge 1} & E_n \wedge X
\end{array}
$$

which implies that the action of $\mathbb{G}_n$ is compatible with the $(E_n)_*$-module structure,

$$g(ax) = g(a)g(x).$$

Definition 6. *A Morava module M is a complete $(E_n)_*$-module with a continuous $\mathbb{G}_n$ action such that, for $x \in M$, $g \in \mathbb{G}_n$ and $a \in (E_n)_*$*

$$g(ax) = g(a)g(x).$$

Example 1. *Let X be such that $K(n)_*X$ is concentrated in even degrees, then $(E_n)_*X$ is complete hence it is a Morava module.*

Example 2. *The module $(E_n)_*E_n$ is a Morava module. Indeed*

$$K(n)_*E_n = (E_n)_*(K(n)) = BP_*(K(n)) \otimes_{BP_*} (E_n)_*.$$

The Atiyah-Hirzebruch spectral sequence collapses for $K(n)_*$, hence $BP_*(K(n))$ is in even degrees and so is $(E_n)_*$. The groups $\mathbb{G}_n$ acts on the left factor of

$$(E_n)_*E_n = \pi_* L_{K(n)}(E_n \wedge E_n).$$

The next goal is to identify $(E_n)_*E_n$. I will unpack the following theorem:

Theorem 3. *Let $K(n)_*X$ be concentrated in even degrees, then there is an isomorphism of Morava modules*

(3)
$$(E_n)_*(E_n \wedge X) \simeq \mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*X).$$

In particular,

$$(E_n)_*E_n \simeq \mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*).$$

To make sense of this theorem, I need

(i) a Morava module structure on $\mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*X)$.
(ii) a map from $(E_n)_*(E_n \wedge X)$ to $\mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*X)$ compatible with this structure.

For such X,

$$\mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*X),$$

denotes the continuous maps from $\mathbb{G}_n$ to $(E_n)_*X$, where $(E_n)_*X$ has the topology induced by $\mathfrak{m}$. This is complete with respect to the ideal $\mathfrak{m}$. Further, it has a natural $(E_n)_*$-module structure: if

$$\phi : \mathbb{G}_n \to (E_n)_*X,$$

is an element of $\mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*X)$, and $a \in (E_n)_*$, then, for $h \in \mathbb{G}_n$, let

$$(a\phi)(h) = a\phi(h).$$

So $\mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*X)$ is a complete $(E_n)_*$-module. There are many choices for the action of $\mathbb{G}_n$ on $\mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*X)$. I will choose that of [1]. For $\phi : \mathbb{G}_n \to (E_n)_*$, let

$$(g\phi)(h) = \phi(g^{-1}h).$$

Now we can construct the map. I do it $(E_n)_*E_n$ but the same construction works if we smash it with $X \xrightarrow{1} X$. Given $x \in (E_n)_*E_n$, define the map

$$\phi_x : \mathbb{G}_n \to (E_n)_*,$$

by setting $\phi_x(h)$ to be the $K(n)$-localization of the composite

$$S^0 \xrightarrow{\ x\ } E_n \wedge E_n \xrightarrow{h^{-1} \wedge 1} E_n \wedge E_n \longrightarrow E_n \ .$$

For this to be compatible with the Morava module structure on $(E_n)_*E_n$, we must have

(4) $$(g\phi_x) = \phi_{gx}.$$

The following diagram gives the map ϕ_{gx}:

$$S^0 \xrightarrow{\ x\ } E_n \wedge E_n \xrightarrow{\ g \wedge 1\ } E_n \wedge E_n \xrightarrow{\ h^{-1} \wedge 1\ } E_n \wedge E_n \xrightarrow{\ m\ } E_n \ ,$$

and indeed

$$\phi_{gx}(h) = \phi_x(g^{-1}h) = (g\phi_x)(h),$$

and the map (3) is an isomorphism of Morava modules. The map

$$\eta_L : (E_n)_* \to \mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*)$$

defined by

$$x \mapsto \eta_L(x)(g) = g^{-1}x,$$

is the left unit. The right unit

$$\eta_R : (E_n)_* \to \mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*),$$

is given by

$$x \mapsto \eta_R(x)(g) = x.$$

Theorem 4. *There is an isomorphism of Hopf algebroids over $\mathbb{Z}_p$,*

$$((E_n)_*X, (E_n)_*(E_n)) \simeq ((E_n)_*X, \mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*)).$$

Remark 3. *A different choice of $\mathbb{G}_n$-action on $\mathrm{Hom}^c(\mathbb{G}_n, (E_n)_*)$ with a compatible isomorphism (3) gives a constant left unit, which is more natural. See [2].*

Dylan is going to show us how the Adam's novikov spectral sequence arises from a simplicial resolution

$$X \longrightarrow E \wedge X \rightrightarrows E \wedge E \wedge X \Rrightarrow \cdots$$

If you take the $K(n)$-local version of this resolution and consider the isomorphism (3) you'll notice that it's computing the group cohomology of $\mathbb{G}_n$ with coefficients in $(E_n)_*X$.

Theorem 5. *The $K(n)$-local E_n-Adams Novikov spectral sequence has E_2-page given by*

$$H^s(\mathbb{G}_n, (E_n)_tX) \implies \pi_{t-s}L_{K(n)}X.$$

This is commonly called the homotopy fixed point spectral sequence.

The machinery used to prove Theorem (4) is used to show the following fundamental theorem:

Theorem 6. Morava's Change of Rings Theorem *Let $I_n = (p, v_1, \ldots, v_{n-1})$. If BP_*X is I_n-torsion, then there is an isomorphism*

$$\mathrm{Ext}_{BP_*BP}(BP_*, v_n^{-1}BP_*X) \simeq H^*(\mathbb{G}_n, (E_n)_*X).$$

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